Overcoming Dyscalculia & Difficulties with Number
Ronit Bird
Praise for the first edition

‘Ronit Bird is one of the most skilled and experienced teachers of learners suffering from dyscalculia. Her approach is based on years of reflective practice but also a deep understanding of the roots of numerical difficulties and disabilities. She stresses the importance of starting with concrete and manipulable materials before moving on to more symbolic materials. Her teaching scheme builds systematically on the basis of the learner’s current understanding, rather than on mechanical measures of performance. This seems to me of fundamental importance.

Overcoming Difficulties with Number provides a wealth of numerical activities and games, taking the most effective from a range of sources, including Cuisenaire rods and domino patterns for the earliest stages where learners are still counting in ones. As learners progress, clear methods for reasoning about more complex numbers are introduced. She provides very lucid methods for areas where many children, not just dyscalculics, have great difficulty, such as solving $5\frac{1}{2} \times 1\frac{1}{2}$ or $(x + 1)(x + 3)$ using grids. I highly recommend this book for teachers and teaching assistants who deal with children who have number troubles, but I also believe that most teachers of early maths will find much that is helpful with all learners.’

Professor Brian Butterworth, University College London
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About the Online Resources

With your purchase of the new edition of *Overcoming Dyscalculia and Difficulties with Number* you have been given access to the Online Resources (OR), available at https://study.sagepub.com/corwin/overcoming2e.

The website contains a large number of resources, allowing teachers to put together a tailor-made package of suitable activities from different sections of the book for particular pupils or groups.

The availability of resources on the website is highlighted in the text with the following symbol: 

**Resources on the website**

Teaching resources to download and print off:

- Appendix 1 Introduction to concrete manipulative materials
- Cuisenaire rods leaflet (print on either side of A4 paper before folding)
- Domino cards
- Digit cards and box
- Multiplication and division word problems
- Skeleton number lines for exploring times table patterns
- *Marching On* game boards
- *Plus or Minus* Game card templates
- Multiples 1–6 game board
- Multiples 4–9 game board
- Name the Factors game board
- More Factors game board
- Component Su Doku puzzles

The Online Resources also include four videos, one relating to each of the Parts I, II, III and IV in the book.
About the author

Ronit Bird is a teacher whose interest in pupils with specific learning difficulties began with a focus on dyslexia. She qualified as a teacher at London University and subsequently gained a further qualification as a specialist teacher. While working with dyslexic pupils in a mainstream school, Ronit started to develop strategies and teaching activities to help support the learning of pupils who were experiencing difficulties in maths.

Ronit has taught in both primary and secondary settings, and has worked as a special educational needs coordinator (SENCO) in both the independent and state sectors. As part of the Harrow Dyscalculia Project in 2006–2010, Ronit ran training courses on dyscalculia for subject leaders, teachers and teaching assistants, later working in an advisory capacity with participating schools across the Harrow LEA.

Ronit currently works as a teacher and continues to create and deliver professional development courses for teachers. Over the past few years Ronit has developed a growing interest in making demonstration videos for teachers and parents.

Ronit can be contacted through her website: www.ronitbird.com
Introduction

This book is for anyone working with learners who have not yet gained a secure understanding of the key ideas behind addition, subtraction, multiplication and division. It is particularly targeted at older learners who could feel humiliated or embarrassed by having to work with material obviously intended for young children but who still need help with some of the fundamental numeracy concepts.

Many of the problems experienced by older learners with dyscalculia, as well as by those who struggle with maths for other reasons, arise from gaps in their basic knowledge. Mathematics is more hierarchical than most other school subjects, in the sense that concepts taught to older students depend on sound foundations laid down in earlier years. This is why there is such a strong focus in this book on putting in place the prerequisite foundation skills. As you will see, I analyse the pre-skills that underpin certain key numeracy topics and provide numerous practical suggestions about how to put in place those pre-skills that are not yet secure. This, in turn, makes the ideas in this book accessible for younger learners as well as those I had originally targeted, including those in the higher years of primary school as well as the lower years of secondary school.

The book identifies a small core of key strategies for numeracy and provides a detailed and practical guide to teaching them:

- Using a knowledge of number components, or number bonds, to avoid having to rely on the damaging habit of counting in ones.

- Bridging through 10, and through multiples of 10.

- Subtraction as complementary addition, so that calculations are always performed working up (forwards).

- Empty number lines for both addition and subtraction, not only as a way of working but also as a way of supporting visualisation strategies.

- Visualisation techniques to support mental calculation.

- The area model of multiplication and division.

- Multiplication and division taught side by side, right from the start, with both operations performed by working up (forwards).

- Using logic and reasoning to extend knowledge and proficiency.
In developing the approach featured in this book, I have deconstructed the essential teaching points of the numeracy strategies listed above and teased out a structured and logical sequence for teaching and learning. The resulting series of teaching activities provides for a systematic and cumulative progression in very small incremental steps, frequently reinforced. In this way, pupils can be introduced to a single new idea at every step, without being prematurely exposed to problems beyond their level of understanding. Pupils are therefore able to experience success while developing their mathematical understanding.

I make no attempt to devise a formal or prescriptive teaching programme. Instead, I have tried to record, in as detailed and accessible a manner as I can manage, ideas that I have found to be successful with my own pupils in combating a range of common misconceptions and difficulties. The work is predominantly practical and oral. I prefer to avoid worksheets because I suspect that having pupils work through pages of written examples will only result in the pupils reinforcing the same inefficient strategies and bad habits that have contributed to their lack of progress.

I strongly believe that the teacher is each pupil’s most important resource and that the teaching focus should always be on teaching for understanding by developing logical thinking and mathematically sound cognitive models.

The activities and ideas in this book are all ready to use with a minimum of preparation. The only equipment needed is what is commonly available in the classroom or easily purchased: dominoes, dice, playing cards and number cards, Cuisenaire rods and base-ten blocks, paper and pencil. Various other resources, including game boards and information about Cuisenaire rods, can be found and printed off from the accompanying Online Resources on the book’s website.

The contents of the book are organised into five parts:

- Part I – Stop counting in ones.
- Part II – The bridging technique.
- Part III – The area model of multiplication and division.
- Part IV – Reasoning strategies.
- Part V – Games collection.

**How to use this book**

The first four parts of the book are set out sequentially, with teaching activities presented in later parts designed to rest on numerical understanding developed at an earlier stage. For example, in order to be able to use the bridging technique (Part II) or to learn efficient strategies to solve multiplication and division problems (Part III), children must have already progressed beyond the stage of relying on their fingers to find simple addition and subtraction answers (Part I).

Each chapter begins with an overview of the topic that puts the teaching points into context. Each chapter also includes a summary of the individual steps that are later expanded in detail.
Introduction

The ideas in the first four parts of the book are structured into a carefully managed sequence of teaching activities designed to be teacher-led. If you are looking for activities and games for parents who are willing and able to support their own children at home, you can find plenty of suitable ideas in the three chapters devoted to putting in place all the necessary pre-skills (Chapters 1, 2 and 5).

Part I addresses one of the most common obstacles to struggling pupils’ progress in numeracy, namely their tendency to rely on counting in ones. This topic is dealt with in a great deal of detail in my previous book The Dyscalculia Toolkit and in the two Exploring Numbers titles of my ebooks, which is why Part I of this book is relatively short with a focus on practice, rather than teaching, of number components. Once pupils have been taught how to free themselves from the ‘counting trap’ they will still need plenty of opportunities to consolidate what they learned, for which purpose a whole range of teaching games and puzzles are provided for practice. A table listing the recommended games against their main teaching points can be found at the end of Chapter 1.

Although it may take a long time for children to completely outgrow their inclination to count in ones, this need not hold them back from beginning to work on the more challenging ideas set out in Part II, at the same time.

Part II contains a detailed step-by-step guide to teaching the bridging technique for both addition and subtraction. Following an analysis of the requisite pre-skills together with suggested activities for teaching these pre-skills to pupils for whom the concepts are not yet secure, two further chapters are devoted to the teaching of bridging through 10, and then through multiples of 10. These chapters advocate the teaching of subtraction as complementary addition, and explore how pupils can move from the concrete stage to the purely abstract stages of mental calculation.

Part III contains a detailed step-by-step guide to teaching the area model for multiplication and division. The first two chapters address both operations together and include an analysis of the requisite pre-skills complete with suggested activities for teaching these pre-skills to pupils for whom the concepts are not yet secure. Two further chapters are devoted to the teaching steps through which pupils can learn to manage the transition from the concrete stage to the abstract stage of understanding the standard written algorithms.

Part IV focuses on reasoning strategies. Pupils with difficulties in maths are rarely flexible in their thinking and must be explicitly taught how to use the few facts they know to derive new facts.

Part V contains all the games recommended in the book, collected together for convenience and ease of access.

The Online Resources accompanying this book contain more than 30 pages of printable resources. They include an appendix about Cuisenaire rods and other concrete teaching equipment, together with various resources such as all the games boards and digit cards required for the games and puzzles.

Access to the four videos made to accompany this new edition is via the OR ".
What’s new in this edition

- The whole of Part I has been reorganised to improve the clarity and flow of the teaching ideas. All the recommended games, including several new teaching games, are collected together in a new section at the end of the book (Part V). A new table has been added to Chapter 1 in which all the games that target component knowledge are listed against the specific teaching points they are designed to address.

- Part III, dealing with the area model of multiplication and division, has been greatly expanded. Many more activities have been added to Chapter 6, while other sections have been rewritten or reconfigured in order to provide more guidance about the teaching of division, a topic that so many dyscalculic learners find particularly difficult.

- Extra illustrations have been added and all the existing illustrations have been reformatted to improve consistency and clarity.

- Easier access to the teaching resources is now available. All the extra material accompanying the text, such as digit cards, games boards and the 25 Su Doku puzzles, can now be accessed online and printed off from the OR.

- Four new videos, one for each of the four teaching sections of the book, have been created for this new edition and can be accessed from the OR.

Specific learning difficulties affecting numeracy

Pupils underachieve in maths for all sorts of reasons, one of which may be the presence of dyscalculia or one of the other specific learning difficulties that affect numeracy. Precise statistics vary, not least because there is a good deal of overlap between the learning difficulties classified under the umbrella term SpLD, but it is estimated that approximately 5 per cent of the general population have dyscalculia and that up to 10 per cent of the general population have one or more specific learning difficulty such as dyslexia or dyspraxia. The prevalence of specific learning difficulties suggests that teachers should expect to find one or two pupils in every average class for whom maths will be a struggle.

It is important to note that specific learning difficulties are found in learners across the whole range of cognitive ability.

Whatever the cause of their difficulty, pupils who have not mastered basic numeracy concepts will find it impossible to make satisfactory progress at secondary school without specialised teaching to address their particular problems.

Dyscalculia

Developmental dyscalculia was first recognised by the Department for Education and Skills (DfES, The National Numeracy Strategy. 05/12/2001, p. 2) in 2001 and defined as:
Introduction

A condition that affects the ability to acquire arithmetical skills. Dyscalculic learners may have difficulty understanding simple number concepts, lack an intuitive grasp of numbers, and have problems learning number facts and procedures. Even if they produce a correct answer or use a correct method, they may do so mechanically and without confidence.

During the 20 years since this definition was first formulated and published, the UK government has not altered or replaced it.

Dyscalculia is neurological in origin and results in difficulties in learning about number and arithmetic. One of its main features is a marked difficulty in distinguishing or processing numerical magnitudes, especially when the quantities are represented symbolically.

As a teacher, you might suspect that you have a dyscalculic pupil in your class if an otherwise competent student has a surprising level of difficulty with ordinary numeric operations and relies on finger-counting, often for all four arithmetic operations, well beyond the age at which most of the others in the class have progressed to more efficient strategies. A dyscalculic learner stands out as having no ‘feel for numbers’ at all, no ability to estimate even small quantities, and no idea whether an answer to an arithmetic problem is reasonable or not. Memory weaknesses, both long-term and short-term, are a great handicap and result in a pupil with dyscalculia being unable to remember facts and procedures accurately, or consistently, no matter how many times they try to learn them by heart. Pupils who have dyscalculia simply cannot remember their times tables reliably, and you may find they can recall some facts one day but not the next. They are also likely to lose track of what they are doing when attempting any procedure that requires more than two or three steps. Even basic counting can be a problem for pupils with dyscalculia, especially counting backwards.

Indicators for dyscalculia are:

- An inability to subtitise (perceive without counting) even very small quantities.
- An inability to estimate whether a numerical answer is reasonable.
- Weaknesses in both short-term and long-term memory.
- An inability to count backwards reliably.
- A weakness in visual and spatial orientation.
- Directional (left/right) confusion.
- Slow processing speeds when engaged in maths activities.
- Trouble with sequencing.
- A tendency not to notice patterns.
- A problem with all aspects of money.
- A marked delay in learning to read a clock to tell the time.
- An inability to manage time in their daily lives.
Dyslexia

A dyslexic pupil might show many of the same indicators as those mentioned above, because it is thought that at least half of all dyslexics also have difficulties with maths. Outside the maths classroom, you might suspect that pupils are dyslexic if they read and write much less willingly and fluently than you might expect, if they read and reread written material with little comprehension and if their spelling is particularly weak, inconsistent or bizarre. Dyslexic learners show much greater ability and understanding when speaking than you could ever guess from looking at the scappy and minimal amount of written work they produce. Other indicators are memory weaknesses, problems with processing auditory information, and difficulties with planning and organisation.

Dyspraxia

A typical dyspraxic pupil does not seem to have the same long-term memory problems as a dyslexic and so might be able to remember times tables facts with ease. Dyspraxia, also known as DCD (developmental coordination disorder), mainly affects motor control, which results in pupils being clumsy and uncoordinated, poor at planning and organisation, and unsuccessful at subjects like PE and sports that require balance and coordination. Dyspraxic pupils cannot process sensory information properly and are therefore forever tripping and falling, dropping and breaking things, and mislaying their belongings. In the maths classroom, dyspraxic pupils have particular difficulty handling equipment such as a ruler, a protractor or a set of compasses, and their written work is likely to be very messy and difficult to decipher.

Diagnosis

A quick and informal way of identifying pupils who need extra help, or further assessment, is to: (a) find whether pupils have difficulties counting backwards, (b) discover which pupils cannot remember times tables reliably, and (c) notice which pupils have no calculation strategies beyond counting in ones. A less subjective identification can be achieved by using Brian Butterworth’s computer-based Dyscalculia Screener, published under the nferNelson imprint and obtainable through GL Assessment. The Screener, which is based on Professor Butterworth’s neuroscientific research, can be administered to several pupils at once and produces a profile of each pupil that can provide evidence (or an absence of evidence) of dyscalculia. A formal diagnosis of dyscalculia can only be given by a qualified educational psychologist after a thorough assessment.

How to help pupils who have difficulties with numeracy

The following principles, exemplified in this book, summarise my approach to teaching learners with dyscalculia:

- Start with concrete materials, making sure that the equipment you use is mathematically sound and is robust enough to model a wide range of numeracy topics. In my opinion, the best concrete resource is a collection of base-ten materials, such as Cuisenaire rods, supplemented by Dienes blocks.
Introduction

- Allow the pupils to use the concrete materials themselves. Do not appropriate them solely for demonstration purposes.

- Never allow the concrete materials to be used mechanically, simply to find an answer. Their value lies in the way they can be used to support visualisation techniques and to build cognitive models.

- Target pupils who are using counting as their only calculation strategy. Before mathematical progress can be made, pupils must be helped out of the ‘counting trap’ by learning to think in terms of components, or chunks, for building or partitioning numbers.

- Allow plenty of time. This means allowing pupils as much time as they need to use concrete materials and to experiment with them. It means building into your teaching plenty of opportunities for recap and revision. It means pausing after asking a question so that a pupil has enough time to think about what the question means without feeling rushed, and yet more time to come up with a reasoned answer. It means addressing problems and misconceptions not at the end of the lesson or the end of a topic, but when there is still time enough for pupils to reconstruct their understanding.

- Think carefully about how much new material and how many topics pupils are expected to learn during the course of a single term. Adjust the syllabus where necessary to allow pupils to become thoroughly familiar and secure with one topic before moving on to the next.

- Engage in a lot of talk as you work. Encourage the pupils to explain what they are doing at every stage, even if they are only talking to themselves. Self-explanation is a powerful and productive learning technique. Get pupils into the habit of reflecting on what they do or see and on putting their thoughts into words. Use and teach a wide variety of appropriate vocabulary.

- Focus on practical activities and teaching games. Apart from the fun that is to be had from solving puzzles and playing games, let your pupils see that maths is something we do, not something we necessarily need to write down. Introduce written calculation as a way of recording only what the pupil has already done concretely and/or has already understood.

- Progress in very small steps. Break down the teaching and learning of every topic into tiny incremental steps and address only one new idea at a time.

- Aim to move pupils gradually from the concrete stage through the diagrammatic stage before moving to the purely abstract stage of calculation. This approach, sometimes abbreviated to CPA, was first developed by Jerome Bruner in the 1950s and 1960s and is now accepted as good practice by maths educators across the world.

- Promote visualisation techniques for mental strategies and explain to pupils exactly how and when to try them, while providing plenty of opportunities to practise visualisation skills.
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- Be sensitive to cognitive load issues. Lighten the burden on pupils’ working memory by encouraging and reminding them to minimise the number of steps in any calculation.

- Give more attention to how a solution is reached than to what the solution is. Try not to make any assumptions about what your pupils already know and ask them, often, to explain how they worked out an answer and how they knew what to do.

- Let pupils make mistakes. Encourage them to see errors as not only inevitable but also a helpful part of the learning process.

- Teach each new strategy by building on a foundation of what is already known. Check that all the necessary pre-skills are secure. Make the connections explicit.

- Allow informal calculation strategies to replace standard written algorithms provided that pupils can consistently reach the correct solution in a reasonable amount of time.

- Minimise the amount that pupils are expected to commit to long-term memory by focusing on key strategies, i.e. those having the widest application. If there are several acceptable ways of tackling or recording a calculation, do not expect your pupils to become familiar with them all. Instead, allow individual pupils to choose whichever method they are most comfortable with and encourage them to practise that method consistently.

- Teach reasoning strategies explicitly. Show pupils how to use logic and reasoning to extend their knowledge of facts and procedures.
Below are sample pages from each chapter of the book.

The extract shown above runs from the start of the book to the end of the Introduction.
CHAPTER ONE

How to help pupils stop counting in ones

Overview

Many pupils who struggle with arithmetic have a tendency to count in ones. This matters because what is a normal stage of development for most children can become a crutch for pupils with poor number sense. Pupils also continue to rely on this unsophisticated and laborious strategy well beyond the stage when it is no longer appropriate or efficient. They have fallen into the ‘counting trap’.

The counting trap is the situation in which pupils know very few arithmetic facts for certain, and therefore have to calculate every new fact from scratch. They calculate by counting in ones, an arduous and long-winded process that puts a great deal of strain on working memory. One reason is that calculating by counting involves a demanding double-counting process: not only must the pupils keep track of the total, they must also keep a separate, parallel count of how much they are adding or subtracting so that they know when to stop the count. The result of finding a solution after too great an investment of effort and concentration is that the answer becomes dissociated from the question and therefore cannot be added to the store of known facts. Which, in turn, means that very few number facts can be instantly recalled or relied upon, so that every new fact must be calculated afresh . . . and so the vicious cycle continues.

In order to help such pupils make progress, it is essential to teach them how to replace their ones-based approach with chunking techniques. The aim is to minimise the number of calculation steps in order to increase a pupil’s chances of achieving a correct solution in a reasonable amount of time and without putting any undue strain on working memory. Professor Eddie Gray coined the phrase ‘the counting trap’ to highlight the way in which a reliance on counting inhibits flexibility. He argues that in order for newly calculated facts to be laid down in long-term memory the counting process must be compressed and that, if this does not happen, learners who fall back on counting procedures actually end up doing a more difficult kind of arithmetic than those who perform more successfully.

A broader issue is what cognitive psychologists call the Cognitive Load Theory. In simple terms, every person has a finite amount of attention and memory at their disposal at any one time. This means that whatever effort and attention is required for a particular task automatically depletes that learner’s reserves, which may leave them without any capacity to process or absorb anything else. In other words, learners risk cognitive overload if too much of their mental bandwidth has to be devoted to one idea, or one aspect of a task (for example, having to laboriously count every single number, one by one, during an arithmetic calculation), leaving little or no spare
Component games: Main teaching point

<table>
<thead>
<tr>
<th>Game</th>
<th>Description</th>
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<tbody>
<tr>
<td>Pelmanism</td>
<td>Numbers 1–10 built from key component facts</td>
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<tr>
<td>3-in-a-row Key Components</td>
<td>Numbers 1–10 split into key components</td>
</tr>
<tr>
<td>Domino Cards Solitaire</td>
<td>Numbers 1–10 built from key component facts</td>
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<td>Make a 1–10 Sequence</td>
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<tr>
<td>Pick-Up Addition Race</td>
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<td>Pick-Up Subtraction Race</td>
<td>Subtracting as finding the difference</td>
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<td>Round the Spot</td>
<td>Numbers or even; adding small quantities</td>
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<tr>
<td>Centennial</td>
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</tr>
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<td>Triples Addition</td>
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<tr>
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<tr>
<td>Who Has the Last Word?</td>
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<tr>
<td>The 3-Component Challenge</td>
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<td>Full House</td>
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<td>Tens Away</td>
<td>10 built out of a pair of components</td>
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<td>Eleven Up</td>
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<td>12 built out of a pair of components</td>
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<td>Thirteen &amp; Fifteens</td>
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<td>Standing Aces</td>
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<td>Pyramid Solitaire</td>
<td>13 built out of a pair of components</td>
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<td>Prisoners</td>
<td>11, 12 and 13 built out of pairs of components</td>
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<tr>
<td>Pontoon</td>
<td>Adding small numbers up to a total of 21</td>
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<tr>
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<td>Shut the Box</td>
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<td>Banking Tens</td>
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<tr>
<td>Marching On</td>
<td>Complements; step-counting in 10s</td>
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<tr>
<td>Conjure the Number</td>
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</tr>
<tr>
<td>Subtract From 15</td>
<td>Subtraction from a teen number, e.g., from 15</td>
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(Continued)
**Overcoming Dyscalculia**

<table>
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<tr>
<th>Minimise the Difference</th>
<th>Subtracting 1-digit numbers</th>
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<tr>
<td>Maximise the Difference</td>
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<td>Magic Squares &amp; Missing Number Puzzles</td>
<td>Components in addition and subtraction</td>
</tr>
<tr>
<td>Component Su Doku Puzzles</td>
<td>Components in addition and subtraction</td>
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A second table, in which all 45 of the recommended games in this book are listed against the equipment required, can be found in [Part 1](#).

A ten-minute video about Part 1, including short demonstrations of the two practical activities in this chapter and some of the recommended games, can be accessed via the OR [link].
Clock the Twelves

This solitaire game provides practice in the components of the number 12. Use a pack of playing cards from which all the Aces and picture cards have been removed, leaving 36 cards, or use a set of four digit cards for each of the numbers 2 to 10 inclusive (OR 🟢).

Rules: Shuffle the pack. Set out 12 cards face down in a circle, as illustrated. Deal out a second round of 12 cards on top, also face down. Begin the third layer by putting a card face down at the top (the 12 o’clock position) and the remaining 11 cards face up around the circle.

Play by clearing away any two cards that add up to 12. Announce the components and the total aloud as each pair of cards is cleared, e.g. 7 plus 5 is 12.

After clearing a card, bring the card beneath it into play by turning it face up. When all three cards in one position have been used, fill the gap with a card from the 12 o’clock position and continue to play. The three cards at the 12 o’clock position may not be turned over in situ, but must be moved to an empty space before the turn come into play. Continue pairing cards for as long as you can, even after gaps appear in the circle.

The game is by, if all the cards are paired up and cleared away.

Thirteen & Fifteens

This variant of a traditional solitaire game practises pairs of components of the numbers 13 and 15, with an inbuilt encouragement to construct the larger number. In this game, Kings count as 13, Queens as 12, Jacks as 11 and Aces as either 1 or 14.
Pontoon

This traditional game, also known as Blackjack, Vingt-et-un or Twenty-one, is a game for three or more players including one who is the dealer. Players should take turns to act as the dealer.

Rules: The dealer shuffles a pack of playing cards and deals two cards to each player face down. Players add their cards, and aim to get the highest score without exceeding 21. Players can score either 1 or 11. Each picture card is worth 10.

The dealer goes round the players in turn who must each say whether they would like another card, in which case they say ‘twist’, or whether they would not like another card, in which case they say ‘stick’. A player whose cards total more than 21 says ‘bust’ and lays the hand face up on the table. A player may ask for a fourth card on the next round, and a fifth or more on subsequent rounds, until all the players have either gone bust or decided to stick. The dealer now turns over his or her own cards and goes through the same process of ‘twist’, or ‘stick’ with the other players looking on.

If the dealer goes bust, all the players who are still in the game win a point. If the dealer sticks at 21 or less, anyone who is closer to 21 wins point. The dealer wins a point if he or she is closest to 21.

If the dealer sticks at the same as another player, with no one else scoring higher, both win a point.

Zero Blackjack

Zero Blackjack is a version of the traditional game described above as Pontoon. All the same rules apply except that it is played with a pack of cards from which all the picture cards have been removed, players are limited to a maximum of five cards each and no one goes bust. In this version of the game all the black cards are regarded as positive and all the red cards as negative. Players aim for a target score of zero to win the round, or win a point for being closest to zero if no player achieves the target.

Shut the Box

This traditional component game for two or three players is often sold in the form of a smart wooden box game, but can just as easily be played with either playing cards or digit cards (OR ☺-) and two ordinary 6-sided dice.

Rules: Each player sets out and controls a set of nine cards, one for each of the numbers from 1 to 9. On your turn, throw both dice, add the two numbers rolled, and turn over any card or any combination of cards from your own set of numbers to match the total, thus removing those cards from play. In the version of the game I am proposing here, you may not match the dice throw exactly unless you have no alternative. For example, if you throw 2 and 3 on the dice, you may turn over the 5 card, or the 1 and 4 cards, but may only turn over the 2 and 3 cards if neither of the other options is available. Continue to throw and to turn over cards until you no longer have cards in play that can match the dice total. Play then passes to the next player.

The winner is traditionally the player with the fewest cards still in play, i.e. still face up. However, for extra arithmetic practice, I recommend asking players to calculate their score by adding the numbers on their remaining cards. The player with the lowest score is the winner, which is an incentive to use the larger numbers during play wherever possible.
Component Su Doku

In a Su Doku puzzle, every digit can appear only once in each row and in each column. In a component Su Doku puzzle, the thicker lines enclose different components that add up to a number in the top left corner of the enclosure.

I have created 25 component Su Doku puzzles at different levels of difficulty for you to print off from the OR, including the two shown here and in Chapter 1. These puzzles at the moderate level, like the one at the left, can be solved by a process of elimination, together with an understanding of the limited possibilities for splitting very small numbers into components. To solve the intermediate puzzles, like the one at the right, pupils must also be able to work out the total values in each row and column and use that fact to make deductions. The most difficult puzzles require several different techniques and the versatility to choose the appropriate strategy at each stage of the solution. All the puzzles provide excellent opportunities to practice building up and breaking down numbers into components and should all be solved using logic and reasoning, without resorting to guesswork or trial and error.

These Su Doku puzzles (also reproduced in Chapter 1) are two of the 25 component puzzles, at varying levels of difficulty, that are ready to be printed off from the Online Resources.
2. Components of the whole numbers up to 10

Although the components of the number 10 are the most important, pupils must also recognise the components of all the other whole numbers below 10. Early understanding is fostered by building number patterns from discrete objects, such as counters or nuggets, and manipulating the objects so that quantities below 10 are physically partitioned and recombined (see The Dyscalculia Toolkit and the dice and domino patterns in Chapter 1).

Pupils should not be required to remember all the number bonds to 10 (except the five complement pairs), but should instead be trained to visualise a number by focusing on a pattern so that they can ‘see’ and manipulate its component parts in their mind’s eye.

Some suggested teaching activities are:

- Key component facts about each number up to 10* (see also the domino card activity in Chapter 1).
- Individual numbers built and explored concretely* (see also the Cuisenaire rods activity in Chapter 1).
- Cuisenaire rods Stories of a number.*
- Missing numbers modelled as missing rod sandwiches.*
- Component games played with Cuisenaire rods, dice or dominoes (see Chapter 1 and Part V).

Cuisenaire rods sandwiches can be set up with a missing rod to explore components of a number. These rods should be read as ‘What must be added to 2 to make 8?’ before being written as a missing-addend problem, with the missing number in either position.
CHAPTER THREE

Bridging through 10

Overview

Bridging is the single most useful mental calculation strategy that pupils can learn.

Bridging through 10 is a technique based on a linear understanding of the number system. Addition is performed by movement along a number line and by adding components in convenient chunks. The number line can, at first, be an actual straight edge, such as a ruler, and later a sketch of a blank line on paper. Later still, especially after sufficient practice, pupils should be able to visualise an imaginary empty number line on which to perform mental addition. The crucially important feature of the bridging through 10 technique is that the addition is not performed one step at a time by counting on ones, but is performed in just two jumps with the number 10 functioning as the stepping stone between the two jumps.

My own experience of teaching bridging on a number line leads me to incorporate a further crucial feature: movement along the line is always in the forward direction.

Although bridging is an abstract concept, it should be introduced concretely by modelling addition with materials that are continuous rather than discrete, namely Cuisenaire rods, before pupils learn to record the work with paper and pencil on an empty number line.

An empty number line is one on which no numbers are labelled in advance. Pupils mark only what is necessary on the empty line as they work through the process of calculating a solution. I do not follow the common practice of preceding every number above the line by a positive or negative sign or of indicating the direction of movement of each jump. Because pupils with specific maths difficulties find it virtually impossible to work backwards, I teach all my pupils to think and work in the forward direction. On a number line, this means that calculations involving positive numbers are always recorded and read from left to right, removing the need for either signs or arrows.

The bridging technique should not be confined to addition problems. Bridging is particularly useful as a way of subtracting by means of complementary addition. Many teachers assume that, because addition can be thought of as movement from left to right along a number line, it is only logical to view subtraction as a movement in the opposite direction. However, most pupils who struggle with maths simply find it too difficult to work backwards, and should never be forced to do so. If pupils can manage one or two small backwards steps mentally, for a calculation such as 26 – 6, or by extension 26 – 8, I would encourage them to continue practising this useful skill. But a backward movement of more than two steps puts too great a burden on the working memory.
Activity 1

Add two 1-digit numbers. Keep the second addend the same; for example, add 6 to various numbers.

Start with the number 6, say, and have the pupils make a rods Story to explore all the component pairs that add up to 6, arranged in a logical order.

6 [dark green]

A Story of 6: sandwiched between two dark green rods are all the possible pairs of components, arranged in order.

Keep the Story of 6 in mind when adding 6 to different numbers using rods, making sure that the examples you choose will require bridging (i.e. add 6 to a number greater than 4).

For example, for $8 + 6$, take a brown rod and a dark green rod and put them end to end.

Example: $8 + 6$

8 [brown]       6 [dark green]

The result is longer than one orange rod but shorter than two orange rods, as can be seen by lining up one or two orange rods alongside the brown. This tells us that bridging will be required and that the answer will be in the teens.

Example: $8 + 6$ (continued)

8 [brown]       6 [dark green]

10 [orange]

Looking at the rods, pupils can see that, in order to bridge through 10, the 6 must be split into 2 and 4, because 2 is the complement of the first number, 8; and 4 is what is left of the 6 after establishing that the first component must be 2.

Pupils can measure the two smaller rods against the dark green rod to prove to themselves that the rods have the same value, or can refer back to the Story of 6 they made at the start. Pupils substitute the red and purple rods for the dark green.

I usually leave the orange rod in place when working through these kinds of examples, to emphasise the fact that we are bridging through 10; however, the orange rod can be removed as soon as the exchange has been decided upon, if pupils find its presence confusing.

(Continued)
Activity 19

Use complementary addition and successive bridging to subtract a 2-digit number from a 3-digit number.

Problems to choose for this step are those that require successive bridging, i.e., problems in which bridging is needed both for the units (bridging through 10, or a multiple of 10) and also for the tens (bridging through 100, or a multiple of 100).

Examples of subtractions that require bridging techniques to be used successively are 246 – 78 or 812 – 43. These problems can be solved in only three steps along an empty number line, as shown here.

Examples: 246 – 78 and 812 – 43

However, many problems that look similar to the ones illustrated above produce jump sizes that are awkward to handle. For example, 275 – 53 ‘solved’ on a number line leaves an addition at the end (175 + 40 + 7) that would require yet more bridging for both the tens and units. By contrast, the same subtraction problem set out and worked in columns is very straightforward because it can be solved by partitioning without any decomposition being required.

In fact, it is worth noting and pointing out explicitly to pupils that the simpler a subtraction problem is to solve by complementary addition along a number line, the more complicated it is to solve by using the standard algorithm in vertical columns, and vice versa.

Example: 275 – 53

Some multi-digit subtractions are easier to solve in columns than by bridging on a number line.

For those subtraction problems that require decomposition, especially successive decompositions, complementary addition on a number line is far easier than vertical subtraction in columns. The bridging technique on an empty number line is exceptionally helpful when numbers are subtracted from multiples of 100 or 1000, as they often are when working with metric measurements.

Example: 2000 – 53

Bridging on a number line is much easier than a column subtraction with successive decompositions.

See also Chapter 9 for a compensation method for column subtraction from multiples of 100 or 1000, by reasoning that 9 is almost 10.

This is one of the activities demonstrated in the video for Part II, accessed via the QR code.
CHAPTER FIVE

Pre-skills for the area model of multiplication and division

Overview

Multiplication can be modelled by repeated addition. Whereas addition can deal with combining different quantities, multiplication deals with situations in which the same number or quantity is replicated several times over. This gives rise to repeating patterns that allow the majority of pupils to memorise those multiplication facts that recur most often, i.e. the facts up to $10 \times 10$. However, pupils who have weak memories and a tendency not to notice patterns are unable to memorise all the times tables facts reliably, and need to be given the tools to derive the necessary facts for themselves.

Division should be presented as the inverse of multiplication, but not as repeated subtraction. Most pupils, even those who are good at maths, find it appreciably easier to work forwards than backwards; pupils with specific maths difficulties are simply unable to count or reason backwards more than one or two steps. Therefore, it makes sense to teach division to all pupils as a process that can be worked in the forward direction, a process by which a number is scaled up or a quantity is repeatedly added, until a specified target number is reached.

By the time pupils start secondary school, there are certain pre-skills that should have been thoroughly understood and internalised. This chapter offers some teaching suggestions for those older learners who have not yet mastered these essential skills and concepts. If your pupils' parents are willing and able, this chapter is a good source of ideas for activities and games to practise at home.

A seven-minute video about the four chapters in Part III can be accessed via the OR.

Summary of necessary pre-skills for the area model of multiplication and division

1. Step counting.
2. Simple mental addition, including sums that require bridging.
3. The ability to see numbers as complete units, not only as collections of ones.
   (Continued)
The Area Model of Multiplication and Division

13× table or the 15× table, so that they can see for themselves how comprehensive and wide-ranging their learned strategies are.

After a while, be sure to mix up your questions so that pupils get enough practice in switching their focus between steps and tables in order to make clear-sighted decisions about the best way to solve each problem. The most efficient route for one person may not be everybody’s preference. For example, deriving 6 × 3 can be achieved by starting from 6 × 2 and mentally adding 6, i.e. a route that some learners will find easier than starting from 5 × 3 and adding 3 to 15. Another example is the tables fact that is traditionally regarded as the hardest of all, 8 × 7, which can be solved efficiently by starting with 7 and doubling three times in a row (7 ... 14 ... 28 ... 56) or by adding a chunk of 3 × 7 to the key fact 5 × 7 (i.e. 35 + 21 = 56). My personal preference is to add the key 2× and 5× facts from the 8× table, because I happen to know that adding the products will be easy (i.e. 40 + 16 = 56).

**Example: 7 × 8 or 8 × 7**

![Diagram of Area Model]

Three efficient ways to calculate the answer to 7 × 8 (or 8 × 7).

Rather than using worksheets to get as much practice as they need, give pupils dice or digit cards to generate random numbers that they must multiply, with or without the support of rectangle sketches. For example, have them roll a 4–9 die at the same time as a 1–10 die, multiply the numbers together then check their answer on a calculator. Or provide self-correcting cards with a multiplication question on one side and the product on the other side. Another way to practise finding a selection of multiplication facts from various tables, in a random order, is to play the two Multiples Games provided in Part V. Both games are played on boards that can be printed off from the OR ⑦⑧.
Activity 18

Use rectangle sketches for division, not just for multiplication.

One of the benefits of the area model, and of recording tables facts on rectangle sketches, is that the same model and the same sketch can represent both multiplication and division at the same time. In this activity, pupils are going to focus on division.

For a problem such as $49 \div 7$, no learner at this stage should be trying to recite the $7 \times$ table one step at a time until $49$ is reached. Instead, they should quickly sketch an outline of the table and practise reasoning from it as follows: We know that $10 \times 7$ is $70$ and $5 \times 7$ is $35$, so does $49$ fall at the beginning (first half) or the higher end (second half) of the table and why? [Answer, the higher end (second half) because $49$ is more than $35$.] How much more is $49$ than $35$; is it more than one step of $7$ away? [Answer: $49$ is $14$ more than $35$, that means two steps of $7$ more than $5 \times 7$.] Therefore seven $7$s are $49$, so $49 \div 7 = 7$.

Here is an example from the $12 \times$ Table: $108 \div 12$. We know that the $10 \times$ and $5 \times$ key facts are $120$ and $60$, so where does the number $108$ fit in, in relation to these? [Answer, it’s at the upper end of the table because $108$ is more than $60$ and, what’s more, it’s much closer to $120$ than to $60$.] How much further than $60$ is $108$? [Answer: $108$ is $48$ more than $60$, which is equal to four more steps of $12$ than the $5 \times$ fact.] Or, we could ask, How far short of $120$ is $108$? [It is just $12$ short. So, $108 \div 12$ is one step less than the $10 \times$ fact.] Therefore, $108 \div 12 = 9$.

Examples: $49 \div 7$ and $108 \div 12$

Even the simplest sketches can help learners reason about division problems.

Encourage pupils to make very simple sketches, marking just the bare minimum of what they need to think about as they work through the logic to find a solution. One way to practise making rectangle sketches to support this kind of thinking is to provide pupils with a whiteboard together with a stack of self-correcting cards that have a division question on one side and the answer on the other. Another way to practise various division facts in a random order is to play the two Factors Games provided in Part V. Both games are played on game boards that can be printed off from the OR.
Activity 2

Sketch a rectangle to model a short multiplication question. Solve by finding the area of separate boxes before combining the partial products.

Make a sketch of $7 \times 13$, as in the example illustrated. Start with one rectangle for the whole $7 \times 13$, then subdivide the 13 into place value components. At first, pupils might want to mark out the sketch the seven rows of 13, to mimic the way the concrete rectangle is built out of rods. As far as possible, encourage pupils to progress to emptier sketches, as shown in the second example, at the right.

Example: $7 \times 13$

![Diagram showing $7 \times 13$ split into smaller rectangles]

Label the areas on the sketch. To find the areas, pupils can be expected to use the multiplication strategies used in the activities detailed in the previous chapter, namely, their chosen strategy for finding times tables facts together with their knowledge of how to multiply a number by 10 or by a power of 10.

Ask pupils to explain in their own words why the two areas, the partial products, need to be combined. If necessary, use a highlighter along the perimeter of the original rectangle – the one that represents $7 \times 13$ – to show that the total area is composed of the two smaller boxed areas. Write $70 + 21$ as an addition problem under the sketch, and have pupils find the total mentally. When partial products are not so easy to combine mentally as the example above, pupils should be allowed to write the addition vertically, if they prefer, and to solve it using column arithmetic, as shown here.

Examples: $8 \times 15$ and $8 \times 27$

![Diagram showing $8 \times 15$ and $8 \times 27$ split into smaller rectangles]

$80 + 40 = 120$

$160 + 56 = 216$

This is one of the activities demonstrated in the video for Part III, accessed via the OR.
Activity 6

Model the standard notation for short division side by side with a concrete example.

While the pupils go through the same procedure as described above, the teacher demonstrates how each step is recorded using the short division standard notation. First, talk about how a short division problem is typically presented and point out to pupils how the notation can be connected to the concrete work with rods. For example, a problem such as $60 \div 5$ is typically presented as follows when it is to be solved by short division: $5 \mid 60$. Point out to pupils how the notation mimics part of the rectangle, showing the top and left sides of the rectangle; how the placement of the number 60 in the notation reminds us that the number to be divided is the area of the rectangle; that the notation tells you that the side of the rectangle must have a height of 5; and that the answer will be the amount along the top of the rectangle.

![Diagram of 5 going into 60]

The short division notation mimics the familiar sketch of the area model of division.

A worked example for the problem $60 \div 5$ might go something like this:

Step 1. Write $5 \mid 60$ on the board and point out, as described in the paragraph above, the connection between the notation and the area model of multiplication and division.

Step 2. Ask pupils to take 60 in rods. The most efficient way is to take six orange rods. Ask if they can begin to make a rectangle that is 5 high out of the tens they have, without making any exchanges. Pupils can begin the division by making a rectangle of five orange rods without exchanging any of the rods. Note that this can only be achieved through the sharing model of division: if the question were interpreted as *How many 5s are in 60?* all the orange rods would have to be exchanged for yellow rods, which would be far less efficient.

Go back to the board. Reminding pupils of the 5-high rectangle they have just built out of five of the six 10s, ask them for the dimension along the top of that rectangle. The answer is 10, or one ten. Write Tens and Units labels in the appropriate place above the written notation, leaving enough room for a written answer beneath the headings, and write 1 in the tens position. Some pupils at this point get muddled between references to ‘ten’, ‘a ten’ or ‘one ten’. For their benefit, you could, at first, write the whole number 10 on the answer line, marking the zero only very lightly on the board, so that another digit can be superimposed onto the zero at a later stage.

Example: $60 \div 5$
Making the Transition to Written Algorithms for Division

This way of setting the work out on paper has clear connections to the box method of multiplication, which pupils have already learned and practised during the work explained previously in Chapter 7. Pupils should practice using diagrammatic sketches on several problems in which 2-digit numbers are divided by a variety of single-digit numbers, and then practice solving the same problems on paper with both the diagram and the standard notation set out side by side. Give pupils a lot of practice as they need to help them fully understand the connection between the semi-concrete, i.e., pictorial, and the abstract methods of solving a short division.

Example: 78 ÷ 6 (continued)

\[
\begin{array}{c|c|c|c|c|c|c}
& & & & & \hline
6 & 78 & & & & \hline
& & & & & \hline
1 & 3 & & & & \hline
& & & & & \hline
8 & 78 & & & & \hline
\end{array}
\]

Activity 8

How to deal with remainders.

This activity follows on from the one above and should not be attempted until pupils understand how to manage simple division problems without remainders. Pupils should also be familiar with the term ‘remainder’ as meaning ‘that which remains’ after a particular stage of the calculation.

Note that remainders in division are first explored towards the end of Chapter 6.

Using concrete materials, start with examples that are similar to the ones already tackled but that will yield a remainder of 1 or 2. After going through the reasoning procedure, pupils will find that they cannot make a neat rectangle from the whole dividend, because there are some rods left over. These rods should be arranged to the right, and be seen as the beginning of a new column. For example, 47 ÷ 4, will require four orange rods to be arranged into a 4 × 10 rectangle, and the black rod to be exchanged either for a purple and a light green rod, or for seven white rods.

Either way, the final complete rectangle measures 4 by 11, but it is immediately obvious that there is a remainder, i.e., an amount that does not fit neatly into the 4-unit high rectangle. In this example, the remainder is 3. But it is also clear from this way of working that the remainder of 3 can be viewed as 3 out of the 4 that would be needed for another full column. Show pupils how to write ‘3 out of 4’ as \( \frac{3}{4} \), or three-quarters.

Example: 47 ÷ 4

\[
\begin{array}{c|c|c|c|c|c|c}
& & & & & \hline
\text{or} & & & & & \hline
\end{array}
\]

\[47 \div 4 = 11\frac{3}{4}\]

(Continued)
CHAPTER NINE
Reasoning strategies

Overview

In a sense, the whole of this book is about reasoning. Certainly, teaching the key strategies in the manner I suggest in the earlier chapters of this book will encourage pupils to think logically and mathematically. Maths is not about arriving at a correct answer by following a recipe, or about acquiring a page-full of ticks in a school exercise book. It is about making sense of the world, investigating ideas and concepts, making connections, developing and strengthening abstract cognitive skills.

If we acknowledge that pupils should be taught for understanding, we must allow them to limit the facts they have to know by heart to a minimum number of key facts. We must also limit the number of strategies that must be learned to only those key strategies that have the broadest applications. To compensate for the lack of automatic recall, we must encourage pupils to use logic and reasoning to derive what they need to find from what they already know. Pupils who struggle with numeracy at secondary school will not yet have discovered this approach for themselves and therefore need explicit teaching about reasoning strategies.

While working on any of the ideas in this chapter, do encourage your pupils to commentate aloud on everything that happens, in their own words. Self-explanation is a powerful learning technique that helps students build knowledge and understanding.

A seven-minute video about the reasoning strategies in this chapter can be accessed via the OR "O:

Summary of reasoning strategies

1. Reasoning about missing numbers.
2. Reasoning from complements to 10 to complements of other numbers.
3. Reasoning from complements to near complements.
4. Reasoning from doubles facts.
5. Reasoning that 9 is almost 10.

(Continued)
Overcoming Dyscalculia

The illustration also shows how concrete materials can be used to show that $16 \times 1$ is equal to $8 \times 2$ which is equal to $4 \times 4$. Having pupils sketch rectangular areas to record the concrete exercise illustrated above will help draw their attention to the fact that if one dimension of the rectangle is halved, the other is doubled. Similar exercises can be carried out with quantities that cannot be easily doubled and halved, for example transforming a $15 \times 3$ rectangle into one that measures $5 \times 9$, or vice versa. The general idea is about conservation of area, so that the area remains constant if the measurement of one of the sides is divided by a certain number while the second measurement is multiplied by the same number.

The implications are that some problems that are presented as long multiplication questions can be quickly turned into short multiplication questions. The benefit can be either fewer steps to calculate, or, more often, easier numbers to work with. Pupils with a secure understanding of the area model of multiplication can, by logic and reasoning, turn awkward multiplication questions into accessible ones.

For example, a problem such as $24 \times 17$ can be sketched on a rectangle and transformed into an equivalent rectangle with $24 \div 4$ on one side and $17 \times 4$ on the other, i.e. $6 \times 68$. This, being a short multiplication question, is already a simpler problem to compute than the original long multiplication of $24 \times 17$. But it can be made even easier by being turned into either $3 \times 136$ or $2 \times 204$.

Pupils can reason about how this calculation method works by performing the necessary multiplication in stages. Pupils are able to record the interim answers on a familiar sketch that carries meaning, by virtue of the previous work they have undertaken on the area model of multiplication. Pupils are left with a relatively simple calculation to make to reach the answer. This method can help pupils sidestep multiplication by numbers above 5, which they may find difficult to remember, by substituting several steps involving multiplication by smaller numbers.