the Dyscalculia Toolkit
Ronit Bird

4th Edition

Age range 6 – 14
220 Activities
55 Games
online resources
the Dyscalculia Toolkit
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4th Edition
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About the author

Ronit Bird is a teacher whose interest in pupils with specific learning difficulties began with a focus on dyslexia. She qualified as a teacher at London University and subsequently gained a further qualification as a specialist teacher of learners with SpLD. While working with dyslexic pupils in a mainstream school, Ronit started to develop strategies and teaching activities to help support the learning of pupils who were experiencing difficulties in maths.

Ronit has taught in both primary and secondary settings, and has worked as a SENCO in both the independent and state sectors. As part of the Harrow Dyscalculia Project in 2006–2010, Ronit ran training courses on dyscalculia for subject leaders, teachers and teaching assistants, later working in an advisory capacity with participating schools across the Harrow LEA.

Ronit currently works as a teacher and continues to create and deliver professional development courses for teachers. Over the past few years, Ronit has developed a growing interest in making demonstration videos for teachers and parents.

Ronit can be contacted through her website: www.ronitbird.com
Online resources

With your purchase of the new edition of *The Dyscalculia Toolkit* you have been given access to the Online Resources, available at: https://study.sagepub.com/corwin/birdtoolkit4e

The website contains a large number of resources, allowing teachers to put together a tailor-made package of suitable activities from different sections of the book for particular pupils or groups.

The availability of resources on the website is highlighted in the text with the following symbol: ©.

Resources on the Website

Teaching resources to download and print off:

- Summary of the main teaching points addressed by the games
- Table of activities and games according to their numeracy topic or main teaching point
- Tracking sheets
- Leaflet: Some things you can do with your Cuisenaire rods
- Dot pattern cards 1–10
- Dot pattern cards 1–10 with box
- Extra large (nugget-sized) dot pattern cards 1–10
- Number track with Slavonic shading
- Digit cards, 0–9 and 1–10
- Digit cards, 1–9 and 11–19
- Digit cards, 1–25
- 1 cm squared paper
- Shallow trays: 2 cm × 10 cm and 10 cm × 10 cm
- The Basic 8 cards
- Place value mat
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100-squares with horizontal or vertical numbering
Times tables squares, complete and blank

Activity sheets to download and print off:
Change one dot pattern into another
Sort and re-sort a set of dominoes
Compare the difference and equalise
Complements to 100
Complements to 100 tracking exercise
Money and change
Double and half
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Times tables on a number line

Game boards to download and print off:
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Collect 5s
Draw Your Race on a Number Line
Key Facts Triad Game
Post-It Note Subtraction
Cover the Number, or Shut the Box
Ten in a Bed
Regroup
It All Adds Up
Five and What’s Left
Race Along a Number Line and Bridge
Race to the End of the Number Line
Four Throws to Reach 100
Race through a 100-Square
Place Value Boxes
Don’t Walk If You Can Take the Bus
Mouse Tables Multiplication
Mouse Tables Division
The Multiples Game
Factors
More Factors
Divisibility Rules

Videos to help demonstrate practice:

Section 1: Using Concrete Materials to Learn Relationships Between Numbers Up to 10
Collect 5s Game
Key Components Guessing Game

Section 2: Extending Earlier Work on Addition and Subtraction to Numbers Above 10
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Section 3: Place Value, Exploring Through Concrete Materials
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Section 4: Times Tables, Multiplication and Division
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Introduction

This book is for teachers who are looking for practical ways to help pupils who struggle with numeracy. It is aimed mainly at primary teachers who do not have a specialist background in either maths or special needs.

I use the word ‘teacher’ loosely to mean anyone who supports children in their learning. Parents, for example, are ideally placed to use the practical activities and games in this book to promote maths as a practical subject full of patterns and puzzles, and therefore full of interest and fun. I hope the ideas in this book will be of interest to any adults who support pupils in junior school and in the early years of secondary school, no matter whether the adults in question are classroom teachers, teachers of numeracy, teaching assistants, parents or specialist staff in special needs departments. Because the suggestions presented here are designed to promote understanding and to help learners make mathematical connections, the ideas in this book can be used to teach the basic principles of numeracy to any learner.

What does this book contain?

Inside this book you will find a collection of teaching activities and games. The activities have been developed over a number of years of teaching dyscalculic, dyslexic and dyspraxic learners, either on a one-to-one basis or in small groups of pupils who have been withdrawn from lessons for extra support. The activities are equally appropriate for children who have been diagnosed with dyscalculia as for those whose difficulties with number arise from other specific learning difficulties such as dyslexia or dyspraxia.

The book is organised into four sections:

- Section 1: Early Number Work with Numbers Up to 10
- Section 2: Basic Calculation with Numbers Above 10
- Section 3: Place Value
- Section 4: Times Tables, Multiplication and Division.
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The philosophy behind my teaching methods is to provide children with the kinds of practical experiences that will help them build sound cognitive models. Because the emphasis is on doing the maths rather than recording it on paper, you will find very few worksheets or ideas for written work in this book. What you will find instead is 220 teaching activities and 55 games. My activities and games are easy to set up and most are ready for immediate use with individual pupils or small groups.

Across the four sections, you will find more than 220 ideas for practical activities that take advantage of concrete manipulative materials as a route to learning and understanding basic numeracy concepts. A small proportion of the recommended ideas — perhaps a tenth — are based on discrete materials such as counters, while the rest all require continuous materials such as Cuisenaire rods. A further 100 activities and many of the games in the book are designed to help learners make the important transition from the concrete and diagrammatic stages towards more abstract mathematical thinking.

Resources for all four sections can be found on the book’s website: https://study.sagepub.com/corwin/birdtoolkit4e. There are 78 pages of resources, ready to be printed off for immediate use. They comprise 20 pages of teaching equipment such as dot pattern cards and digit cards, 16 pages of activity sheets, 22 game boards and 20 pages of summary tables and pupil tracking sheets.

The summary tables, which can be found at the end of this chapter and on the book’s website (https://study.sagepub.com/corwin/birdtoolkit4e), are designed to help you identify the most suitable activities and games for an individual learner or a teaching group. The tables allow for convenient cross referencing, by listing the activities and games against the numeracy topics for which they were devised.

To help teachers track the progress of individual pupils, detailed tracking sheets are provided on the book’s website (https://study.sagepub.com/corwin/birdtoolkit4e). The tracking sheets can be edited before being printed off.

The Appendix contains a summary of the more commonly used concrete materials, including an introduction to Cuisenaire rods. Because I use Cuisenaire rods so extensively in my teaching activities, and have found so many people unfamiliar with their use, I have also included on the book’s website (https://study.sagepub.com/corwin/birdtoolkit4e) a leaflet of practical ideas, written originally for parents.

To accompany The Dyscalculia Toolkit I have created 10 videos with a total running time of 50 minutes, full of explanations and demonstrations. The videos can be accessed and watched as often as you like on the book’s website (https://study.sagepub.com/corwin/birdtoolkit4e). There are four longer videos with commentary, one for each of the four sections in the book, plus six short stop-motion silent movies demonstrating a selection of games across the four sections.

What’s new in this edition?

- Revisions and additions to Section 4: Times Tables, Multiplication and Division. Most of the new content focuses on division, because so many learners find division particularly difficult. For example, there are new ideas designed to help children make the transition from informal
diagrams, used to support their thinking, to more abstract calculation methods including the standard written short division algorithm.

- More activities. Some have been added and others reconfigured, bringing the total to 120 activities across all four sections.
- More games. Five new games have been added, including two board games, two card games and a domino game, bringing the total to 55 games across all four sections.
- More downloadable teaching materials. New additions include digit card templates and two new game boards, bringing the total of printable numeracy resources to 78 pages.
- Content has been added to the video for Section 4 in order to include more on division, bringing the combined running time of all ten videos to 50 minutes in total.
- The pupil tracking sheets have been updated, as have the summary tables that cross reference between the activities or games and the various numeracy topics or calculation strategies they are designed to target.

What equipment will I need?

I have deliberately included activities that require only what can usually be found in a normal maths classroom or can easily be acquired by parents, such as counters, Cuisenaire rods, Dienes blocks, number cards, dice, dominoes, paper and pencils. This equipment, used as directed across all four sections of the book, plays a crucial role in providing children with as much hands-on practical experience as they need in order to develop number sense and mathematical understanding. There is no need to buy special equipment, or commercial games and resources that tend to target only a single topic.

At the end of the book you will find an Appendix all about concrete manipulative materials for maths, including counters, Numicon, Cuisenaire rods, Dienes blocks, Stern materials and various hybrid materials.

Why is there such a strong emphasis on games?

I invariably use a lot of games in my teaching, and not simply because they are fun. Provided that the games are carefully chosen, or carefully designed, to target only a single mathematical idea at a time, games furnish pupils with the opportunity and the incentive to practise the specific techniques that we want them to acquire, allowing the ideas to become habitual and gradually more fluent. For example, having taught pupils the complement facts of 10 – the five number bonds 5 + 5, 4 + 6, 5 + 7, 7 + 2 + 8 and 1 + 9 – by allowing the pupils to manipulate concrete materials as they explore these numerical relationships in an active and practical way, pupils will still need plenty of practice using these five facts before they can become absolutely secure. That is why you will find five games, as well as several activities, that specifically target the complement facts of 10. Learners with specific learning difficulties tend to need much more repetition and rehearsal than their peers, spread over a longer period. But there are only so many times someone can recite the facts, or complete worksheets featuring them, without boredom setting in.
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In parallel with work on key facts, I regularly and explicitly teach pupils reasoning strategies about how to derive an unknown fact from a known and practised fact, such as how to find various steps of a multiplication table from the three key facts for that table. A worksheet is likely to give only mechanical practice in producing the answer because it has been designed as a way of testing automatic recall, whereas a well-designed game will practise the necessary steps required to reach an answer through logic and deduction.

As a teacher, I am constantly aware of the need to contrive situations in which the facts and techniques that my pupils need to master can be rehearsed in as many different ways as possible. Games are extremely valuable in this context because children are naturally motivated to spend time playing games over and over again and rarely notice how much learning and reinforcing is taking place while they are actively and productively engaged in play. As well as being more enjoyable, games are more powerful than worksheets because each time a game is played different challenges might be presented, in a different sequence, leading to new considerations and different outcomes, all of which provides for a much more varied, stimulating and active learning experience.

Purposeful activities and targeted games are central to my approach to teaching learners who have difficulties with basic maths. The activities I specify in this book are never intended to simply back up paper-and-pencil techniques or abstract methods. Neither are my games ever intended as just a bit of fun to fill in the spare time at the end of a lesson. All the activities and games that I design are invented to provide the actual learning experience for a variety of very specific maths topics.

How to use this book

Please do not feel that you ought to start at the beginning of the book and work through to the end, or even to keep to the sequence in which the ideas are presented. You should feel free to pick and choose activities, depending on your pupils and your knowledge of their particular areas of difficulty. Some activities may need to be repeated often, or revisited at regular intervals; others may be valuable to try only once for particular pupils, or not at all. When activities naturally follow on from each other, the text clearly signals the fact. Some activities may need to be preceded by others from another section; for example, some understanding of place value (Section 3) is required before attempting some of the work on larger numbers (Section 2) and before some of the work on multiplication and division (Section 4). Once you begin working closely with pupils, you will find that you are the person best placed to uncover any misconceptions or sticking points that could usefully become the focus of subsequent lessons.

Each of the four sections starts with a short overview, putting the topic of that section into context. Following the overview, you will find a summary of the main problems associated with the topic leading to a list of ideas on how to help. These summaries are presented as bullet points for ease of reference. The remainder of each section is dedicated entirely to the teaching activities and games, set out as clearly and concisely as possible with a minimum of explanatory background or theory. Printable and photocopiable resources from all four sections are provided on the book's website (https://study.sagepub.com/corwin/birdtoolkit4e(2)), making the activities and games accessible and ready to use, with the minimum of preparation.
I have targeted what I know to be specific areas of difficulty and have deliberately broken down the teaching and learning into extremely small steps. Each section is loosely structured in order of difficulty, starting with concrete activities and progressing gradually through learning activities that are designed to help pupils move through the intermediate diagrammatic stage and right up to the abstract stage of calculation.

Most of the activities are designed to be teacher-led, rather than for children to work through on their own. It is important to ask lots of questions, to direct the discussion carefully, to point out any connections with previous activities and other maths topics and to encourage pupils to talk a lot about what they are doing, and why, while they are doing it. Naturally, pupils will do best in an atmosphere where mistakes are regarded as a normal, and even an instructive, part of the learning process.

Whether inside the classroom or at home, the best results will be achieved by frequent, regular, short but unhurried sessions, each of which should include a variety of activities and topics and a sensitive balance between revision and new content. Daily sessions will soon improve pupils’ attitude and will steadily boost their self-assurance, their sense of achievement and their maths performance.

The 220 activities are each labelled according to the main teaching point they have been designed to address. A list of teaching points is also included in the instructions for each of the 55 games. The main numeracy topic addressed by each game is summarised in the first of the two tables at the end of this Introduction. The second summary table at the end of this Introduction links the activities and games in this book to a list of numeracy topics and teaching points so that you can easily find ideas to target a particular gap in a pupil’s knowledge or to address a specific misconception or need. As well as appearing at the end of this chapter, both the summary tables can be found on the book’s website (https://study.sagepub.com/corwin/birdtoolkit4e). Tracking sheets, accessible from the book’s website (https://study.sagepub.com/corwin/birdtoolkit4e), are provided in this edition of The Dyscalculia Toolkit. Closely linked to the summary tables, the tracking sheets are designed to help you plan work for individual students or groups of pupils, and to use for tracking and recording a learner’s progress. The tracking sheets can be edited before being printed off.

**What is dyscalculia?**

Developmental dyscalculia was first recognised in the UK by the Department for Education and Science in 2001 and defined as: ‘a condition that affects the ability to acquire arithmetical skills. Dyscalculic learners may have difficulty understanding simple number concepts, lack an intuitive grasp of numbers, and have problems learning number facts and procedures. Even if they produce a correct answer or use a correct method, they may do so mechanically and without confidence’ (DfES, *The National Numeracy Strategy*, 0512/2001, p. 2).

There is a debate about whether true dyscalculia differs from the maths difficulties experienced by some dyslexic and dyspraxic learners, a debate I am happy to leave to the academics. What
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matters to me is the fact that the same sorts of intervention seem to help many pupils who are underachieving in maths, whatever label they have been given. I believe that the coming years will see a growing recognition of the particular problems and educational needs of dyscalculic learners, in much the same way as the last three or four decades have seen an increasing acceptance of the existence of dyslexia and a developing consensus about the best teaching and learning approaches for these pupils.

Research into dyscalculia is still at an early stage, but it is estimated that dyscalculia affects roughly 4–6% of the population. This equates to at least one child in any average classroom.

What are the indicators for dyscalculia?

As a teacher, you might suspect that you have a dyscalculic pupil in your class if an otherwise competent student has a surprising level of difficulty with ordinary numeric operations and relies on finger-counting, often for all four arithmetic operations, well beyond the age at which most of the others in the class have progressed to more efficient strategies. A dyscalculic learner stands out as having no ‘feel for numbers’ at all, no ability to estimate even small quantities, and no idea whether an answer to an arithmetic problem is reasonable or not. Memory weaknesses, both long-term and short-term, are a great handicap and result in a pupil with dyscalculia being unable to remember facts and procedures accurately, or consistently, no matter how many times they try to learn them by heart. Pupils who have dyscalculia simply cannot remember their times tables reliably, and you may find they can recall some facts one day but not the next. They are also likely to lose track of what they are doing when attempting any procedure that requires more than two or three steps. Even basic counting can be a problem for pupils with dyscalculia, especially counting backwards.

Indicators for dyscalculia include:

- an inability to subitise (see without counting) even very small quantities
- an inability to estimate whether a numerical answer is reasonable
- weaknesses in both short-term and long-term memory
- an inability to count backwards reliably
- a weakness in visual and spatial orientation
- directional (left/right) confusion
- slow processing speeds when engaged in maths activities
- trouble with sequencing
- a tendency not to notice patterns
- a problem with all aspects of money
- a marked delay in learning to read a clock to tell the time
- an inability to manage time in daily life.
What about learners with other specific learning difficulties?

A dyslexic pupil might show many of the same indicators as those listed above, because it is thought that at least half of all dyslexics also have difficulties with maths. Outside the maths classroom, you might suspect that pupils are dyslexic if they read and write much less willingly and fluently than you would expect, if they read and re-read written material with little comprehension and if their spelling is particularly weak, inconsistent or bizarre. Dyslexic learners show much greater ability and understanding when speaking than you could ever guess from looking at the scrappy and minimal amount of written work they produce. Other indicators are memory weaknesses, problems with processing auditory information, and difficulties with planning and organisation.

A typical dyspraxic pupil does not seem to have the same long-term memory problems as a dyslexic and so might be able to remember times tables facts with ease. Dyspraxia, also known as DCD (developmental coordination disorder), mainly affects motor control, which results in pupils being clumsy and uncoordinated, poor at planning and organisation, and unsuccessful at subjects like PE and sports that require balance and coordination. Dyspraxic pupils cannot process sensory information properly and are therefore forever tripping and falling, dropping and breaking things, and mislaying their belongings. In the maths classroom, dyspraxic pupils have particular difficulty in handling equipment such as a ruler, a protractor or a set of compasses, and their written work is likely to be very messy and difficult to decipher.

A pupil with attention deficit hyperactivity disorder may signal his (and it is usually a boy) presence by being unable to stop fidgeting or to sit still, being too easily distracted by outside stimuli, having a tendency to talk and interrupt excessively, and finding it extremely difficult to stay on task and see any undertaking through to the end. I mention the condition here only because nowadays pupils with ADHD or ADD tend to come under the umbrella term of ‘learners with specific difficulties’. However, pupils with attention disorders may not have any specific problems with numeracy or maths once they have found a way to manage their impulsivity and concentration difficulties.

What kind of teaching do dyscalculic learners need?

All numeracy teaching should aim to help learners build up a sound mathematical understanding of numbers and their relationships. The basis of my own teaching approach with dyscalculic learners is to concentrate on numeracy and arithmetic, starting — crucially — with a variety of versatile concrete materials that provide practical experience and strongly visual models. Once a numerical concept has been understood at the concrete level, then, and only then, will I begin to lead the learner gradually but steadily towards some of the more abstract and symbolic methods associated with higher level mathematics.

My own view is that a set of Cuisenaire rods is indispensable for working with dyscalculic learners. I find it the best, most versatile and most powerful tool to offer learners who are struggling to
THE DYSCALCULIA TOOLKIT

build a coherent mental model of the number system. I supplement Cuisenaire rods at the lower end with discrete items, such as counters or nuggets, that can be arranged and re-arranged into dot patterns for the numbers up to 10, and at the upper end with Dienes blocks or other base-10 equipment that can combine with Cuisenaire rods for concrete modelling of 3-digit numbers. One of the great strengths of Cuisenaire is that numbers are not presented as a collection of ones, so that the learner's focus is directed away from counting and towards number relationships. See the Appendix for an introduction to Cuisenaire rods and other concrete materials.

Dyscalculic learners, just like other learners, need to be able to count properly. Counting is, after all, the foundation of all numeracy. But, as soon as counting is secure, children have to be taught calculation strategies that do not rely on counting in ones. For this reason, I recommend plenty of work on building numbers from smaller components, splitting quantities up again into smaller chunks, and recombining the component pieces once more in order to fully explore the composition of numbers and the connection between addition and subtraction. The component work that I describe in this book is less static than simply learning the number bonds because its emphasis is on performing operations on numbers and seeing quantities change as a result of whatever action is being performed.

Working with chunks, or components, rather than ones, is the only antidote to the immature — and damaging — dependence on counting that is so common in pupils who struggle with numeracy. When counting is the only strategy known to learners, they have fallen into the 'counting trap'. See the Overviews of Section 1 and Section 2 for more about this pervasive problem. The only way out of the vicious cycle is to explicitly teach learners calculation strategies based on components, i.e. chunks, and not on counting on, or counting back, in ones.

Similarly, the times tables work and the multiplication and division activities that I recommend in this book are very far removed from the all-too-common practice of giving children a list of tables facts to learn by heart, a situation that leaves many pupils without any idea about what multiplication or division mean or how to use or apply the facts they have been asked to memorise. My teaching approach to tables is based on the area model of multiplication and division, an interpretation that inherently connects multiplication with division from the very beginning and one that can be modelled with Cuisenaire rods to produce rectangular shapes that are easy to read, understand and visualise.

Problems with numeracy often go hand in hand with significant memory weaknesses. This is why simple repetition will never be a way forward for dyscalculic pupils, however hard or often they are drilled. The best way to work around learners' memory problems is to focus on only a few key facts, those that are more important or have the widest application: first allow learners to thoroughly explore and internalise the key facts; then teach them explicitly how to derive whatever other facts they might need by reasoning logically from the key facts they already know.

Visualisation is a strategy that should be explicitly taught to dyscalculic pupils as a route towards mental calculation strategies. Immediately after a session of concrete work, pupils can be asked to close their eyes and try to recreate some of the work in their mind's eye. Diagrammatic calculation methods, such as empty number lines for addition and subtraction or the area model for multiplication and division, can be introduced as a way of recording concrete work with
Cuisenaire rods and later extended to support visualising techniques, thereby creating a bridge between concrete exploration with manipulative materials and the more abstract work that is the norm in mainstream schools. The transition between concrete and abstract work is an important stage that needs to be planned for and should not be rushed.

My final observation about what kind of teaching approach works best for dyscalculic learners is a recommendation to break down every bit of teaching and learning into the tiniest of incremental steps and not to make any assumptions about what pupils already know. For example, just because a child knows, say, that five counters can be arranged into the familiar dice pattern for 5, it does not follow that the same child will know that none of the other dice patterns can be created out of exactly five counters; or just because a child has discovered that adding 1 to each of the numbers up to 10 results in the next number in the counting sequence or that taking 1 away results in the previous number, it does not follow that the same child will know how to add 1 to a 2-digit number, let alone be able to work out how many to take away from a quantity in order to leave 1. Sound numerical understanding can only develop if it rests on secure foundations at every stage. The importance of tightly focused practical activities that address only a single new idea at a time cannot be overestimated.

**What’s next, after working through this book?**

I have written two other books published by Sage that follow on from this one, although each book is complete in itself and can be used independently of the other two. *Overcoming Dyscalculia and Difficulties with Number* (2009) is aimed at learners who are working at a slightly higher level or who have, perhaps, already worked through many of the ideas in *The Dyscalculia Toolkit*. *Overcoming Dyscalculia and Difficulties with Number* analyses some key numeracy strategies – such as bridging through 10, or learning multiplication tables through the area model – in very great detail, setting out step-by-step instructions on how to teach the strategies to learners who find the concepts difficult. The focus is on teaching for understanding while at the same time helping learners make the transition from practical exploration to more abstract and canonical calculation methods.

*The Dyscalculia Resource Book* (2017) is a collection of ready-to-use and printable games and puzzles, all carefully targeted to practise the crucial foundation skills – such as adding and subtracting in component chunks rather than in ones, or deriving new numeracy facts from known key facts – that children who struggle with numeracy need to master before they can make any significant progress in maths. The games and puzzles in *The Dyscalculia Resource Book* are designed to reinforce what has previously been taught at a concrete level – for example through the activities and games in this *Dyscalculia Toolkit* book – and each is accompanied by clear instructions to the supervising adult on how to manage the activity so as to maximise the learning experience.

You are welcome to contact me through my website (www.ronitbird.com) with any feedback about how your children or pupils respond to the ideas in any of my books. On my website you will also find information about dyscalculia, a variety of free games and teaching resources, a list of Top Ten Tips for Parents and details of a series of ebooks aimed at parents in which embedded videos demonstrate each of the recommended teaching ideas featured in the text.
THE DYSCALCULIA TOOLKIT

SUMMARY OF THE MAIN TEACHING POINTS ADDRESSED BY THE GAMES IN THE
DYSCALCULIA TOOLKIT

This table is provided because the name of each game does not always reveal exactly what topic it was designed to target (unlike the main teaching points of the activities, which are spelled out by their titles).

A downloadable version of this table is available on the book’s website: https://study.sagepub.com/corwin/birdtoolkit4c.

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The extract shown above runs from the start of the book to the end of page 11.

Below are 2 sample pages from each section of the book, followed by the Appendix.
Activity

Make a ‘Story’ of a number

Have pupils set out, in a logical sequence, all the ways of making a number out of two components, for example the ‘Story of 7’ as shown below.

Start with the key fact (i.e. the doubles or near-doubles fact) and ask the child to add one complete row, namely two Cuisenaire rods, each time. Make sure the arrangement of the rods is consistent from row to row (i.e. that the child keeps to the same decision about positioning the rods that increase in size either on the left or on the right) so that a zig-zag pattern is created along one diagonal of the rectangle of rods. This distinctive zig-zag makes visible the important fact that, as soon as the size of one of the components gets bigger by one, the other component must become one smaller to compensate, and vice versa. This is the same logic that we used during the regrouping activity with dot patterns.

The reasoning process depends on an understanding of the conservation of number and also on understanding that addition is commutative. Both these important concepts are strengthened by children carrying out this activity often, focusing on different numbers on different occasions.

Here are the key components of 7: 4 and 3. Look at the next line, and see how the first component, 4, increases by one, becoming a 5. Therefore the other component MUST get smaller by one. So, if you know that 7 can be built from 3 + 4, you also know that 7 can be built from 5 + 2, etc.

Once the Story has been built, the rods can be pulled apart so that there is a visible gap between each pair of components. This reinforces the fact that if we know how to build a number out of two components, for addition, we also know how to split the number into two components, for subtraction.
Game
Cover the Numbers, or Shut the Box

A game for one, two or more players.

Teaching points
- The game teaches splitting all the numbers 1 to 12, in many different ways.
- It gives practice in adding and recognizing small numbers to make the numbers up to 12.

Equipment needed
- Either a manufactured version of this game (several, called Shut the Box, are widely available) or my paper version (see OR ©) called Cover the Number, for which you will also need pencils.
- Two 1–6 dice.

Rules
Each player controls one set of the numbers 1 to 9. On your turn, throw two dice and announce the total. Choose one or more numbers from your set that add up to the same total, and remove them from play, by covering them or crossing them out. Continue like this until you throw a total that cannot be matched, or created, from the numbers that remain available. Record your score as the number of numbers left uncovered. Now it is the next player’s turn. The winner has the lowest score after three rounds.

Cover the Numbers (or Shut the Box)

RULES: Throw two dice and announce the total. Choose a number, or a combination of numbers, that add up to the same total and remove it from play by shading the square(s). Continue to throw the dice and remove numbers from the same 1–9 set, until you throw a total that cannot be created from the numbers that remain. Your score is the number of numbers left uncovered in the 1–9 set. (Variation: Score by adding the numbers left uncovered.) It is now your opponent's turn to keep throwing dice and removing numbers from play, until a dice throw cannot be matched by using, or combining, any of the remaining numbers in the 1–9 set. The winner has the lowest score after 3 rounds.

First Player

Second Player

There are enough sets of numbers on this page for 2 players to play 8 rounds.
Activity

Complementary addition, or subtraction by adding

Step 1
Remind pupils of the work they have already done on complementary addition with small numbers (see Section 1). Ask pupils to explain it back to you, and to each other, first by using counters and then on a number line.

The key idea is that the number to be subtracted is taken away from the beginning of the array of counters, or from the start of the line, so that the whole calculation moves in the forward direction.

Remind pupils that they can physically cross out the amount to be subtracted on a number line beginning at zero, and only begin to use an empty number line that does not start at zero when they are completely convinced that both representations model the same problem. For example:

$$19 - 13 = ?$$

Step 2
Give pupils lots of practice in using number lines for problems like the previous activity, where both the numbers in the question are 2-digit numbers, but where the answer is below 10.

Step 3
Ask pupils to use their number line solutions to the problems in the previous step, and to rewrite the problems as an equalising (or difference) problem, using the missing-addend notation, e.g. \(13 + \square = 19\). This part of the activity reinforces the relationship between addition and subtraction. It also shows pupils that, because work on a number line is in the forward direction, there is no need for arrows on the jumps or operation signs for the quantities.

Step 4
Give pupils the same kinds of problems as previously, but without paper and pencil. Pupils must practise putting an empty number line in their mind’s eye and working forwards along it. You should provide the actual question in writing, as it might put too great a strain on a pupil’s memory to have to keep hold of the numbers of the question, while mentally calculating the answer. This kind of activity should not be timed. However, these problems can be answered in only two steps, so check that pupils who are very slow are not finding the solution by counting in ones.
Game
Frame a Subtraction

A game for two or more players. You can find a short demonstration video of this game, and the related Frame an Addition game, on the OR site.

Teaching points

- The game is about subtraction by complementary addition using bridging through 10.
- The game targets those subtractions that arise from using decomposition methods to solve column subtraction of teen numbers minus single-digit numbers.
- The game supports the transition between concrete and abstract work by requiring players to perform the same calculation first with Cuisenaire rods and then on an empty number line.
- During the game, players model the idea that subtraction can be expressed as ‘difference’ and that finding the answer to a subtraction does not necessarily involve working backwards.

Equipment needed

- Cuisenaire rods.
- A frame or tray surrounding a space measuring 2 cm × 10 cm for each player. This can be made of paper or card (see OR site) or a more durable frame can be cut out of craft foam or kitchen sponge.
- A spinner with a base showing the numbers 11 to 15.
- A 4–9 die.
- Paper and pencil.

Rules

Start every turn with an empty frame. On your turn, spin the spinner and take two rods (one of which must be an orange 10-rod) to match the spin. Put the rods inside your frame. Roll the die to find out how much to subtract. If you have enough rods to carry out the subtraction without touching the 10-rod (i.e. 15 minus 4 or 5, or 14 minus 4) that is the end of your turn.

If the solution to the subtraction problem is less than 10, you must demonstrate how to find the answer by exchanging the orange 10-rod for two smaller rods. Make the exchange in such a way that you can subtract – by physically removing – a single rod that matches the number on the die. For example, in order to subtract 7, the orange rod that represents the 10 of the teen number must be exchanged for 7 + 3 so that the black rod can be removed. The answer to the subtraction is found by adding the two rods that remain in the frame.

(Continued)
Activity

Teach the threefold repeating pattern: units, tens, and hundreds

Make explicit to pupils that numbers are written in groups of three. From right to left (i.e. starting with the smaller values and moving towards the larger values) we first see units, tens and hundreds of ones, followed by units, tens and hundreds of thousands, followed by units, tens and hundreds of millions, followed by units, tens and hundreds of billions, etc. (See the introduction to this section.)

Teach pupils to make and label place value columns in sets of three, like those illustrated below. Use thick separation lines to emphasise the threefold grouping. Tell pupils that, when writing multi-digit numbers, the commas indicate the position of the thick separation lines (more on this later).

Reassure pupils that, once they can read and write 3-digit numbers, they can read and write any number (more on this later).

<table>
<thead>
<tr>
<th>Millions</th>
<th>Thousands</th>
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<tbody>
<tr>
<td>Hundreds, Tens, Units</td>
<td>Hundreds, Tens, Units</td>
<td>Hundreds, Tens, Units</td>
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</table>

<table>
<thead>
<tr>
<th>Millions</th>
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</table>

It is common, in classes for younger children, to see the ‘units of ones’ column labelled simply as ‘ones’. This is not wrong but, logically speaking, all the columns are about ones. For example, when we say that the number 2,000 should be read as ‘two thousand’, the answer to the question ‘Two thousand of what?’ is: ‘Two thousand ones.’ For this reason, I prefer to use ‘units’ and ‘ones’ in the way I have explained at the start of this section.

The aim of this labelling activity is to direct the learners’ attention to the logical system of place value column names. Many dyscalculic learners find the threefold repeating pattern surprising and illuminating, once it has been pointed out to them.
Game

Calculator Skittles

A game for any number of players.

Teaching points

▷ The game teaches place value in large numbers.
▷ It also teaches how to use a calculator for subtraction.

Equipment needed

▷ A calculator for each player.

Rules

Choose any 4-digit number that does not include a zero, and enter it into your calculator. You may repeat digits within your chosen number if you like. The teacher calls out any number between 1 and 9 for the players to ‘knock down’. Each player who has that digit on their calculator screen is now allowed to perform one, and only one, subtraction on the calculator, the aim being to replace that digit with a zero. Any pupils who mistake the place value of the digit and do not manage to ‘knock it down’ to zero must live with their mistake and continue with whatever number their calculator now shows. The teacher continues calling out random digits until one or more of the players achieves a zero display on their calculator and is awarded a point. Players then choose new numbers and a new round begins. The winner is the player with the most points after a certain period of time.

Variation 1

Instead of letting children choose, players get allocated a 4-digit number by dealing digit cards or throwing dice.

Variation 2

Use numbers that have more than four digits.

Tips

The teacher can use a die, or a pack of digit cards, to generate the random numbers between 1 and 9. If pupils write down the number they started with, they can demonstrate at the end exactly how they arrived at zero.
Activity

×9 is almost ×10

The 9× step of any table, or the 9× table itself, is an easy one to learn for children who are familiar with Cuisenaire rods and with the area model of multiplication. For example, for 9 × 4, or 4 × 9, make a rectangle for 10 × 4 out of four 10s and show where an imaginary saw could chop off one unit from the end of every 10 rod. Use a single rod to model the quantity that needs to be removed to turn 10 × 4 into 9 × 4. Put the purple rod on top of the orange rectangle to show that the solution to 9 × 4 is 40 − 4. Move the purple rod so that it sits on top of only one orange, instead of straddling them all. This repositioning makes it clear that all but one of the 10s can remain untouched and that only one rod has to be breached. Complement rods can quickly help us find the answer: the complement of 4 is 6, so 40 − 4 = 36.

Ask the child to use Cuisenaire rods for the 9× step of all the tables, in a random order, to demonstrate the reasoning process. Later, see if children can answer some questions without the rods to support their thinking. Later still, some children will pick up on the linguistic pattern:

9 × 6 is 6 less than 60,
9 × 8 is 8 less than 80,
9 × 9 is 9 less than 90,
9 × 7 is 7 less than 70, etc.
Game
Products in a Row

A game for two players.

Teaching points

- The game practises two multiplication tables at a time.
- It highlights the connection between the multiplication tables of a number and the table of its double, e.g. that any other step of the $2 \times$ table is identical to a step from the $4 \times$ table. The same relationship exists between the $5 \times$ and $10 \times$ tables, between the $4 \times$ and $8 \times$ tables, between the $3 \times$ and $6 \times$ tables, and between the $6 \times$ and $12 \times$ tables.
- Both multiplication and division mental processes are used during play, but without mentioning the word 'division'.
- There is a strategic element to this game, by noticing which cards have been used.

Equipment needed

- 24 blank cards (small laminated cards are ideal).
- A wide strip of paper for each player, on which there is space to set out 12 cards in a row.

Rules

Decide which pair of related times tables to practise. On different occasions, play this game for the $2 \times$ and $4 \times$ tables, the $4 \times$ and $8 \times$ tables, the $3 \times$ and $6 \times$ tables (as in the illustration) and the $6 \times$ and $12 \times$ tables.

Write the answers to the first 12 steps of each of the chosen tables on the cards, one product to a card. Make sure the players notice which products appear in both tables, and understand why. Each player prepares a game board by writing the numbers from 1 to 12, in order, on a strip of paper. Space out the numbers so that one card, when placed on the game board, covers only one number. Players should note down on their board which pair of tables is being targeted.

Take turns to take one card from the top of the shuffled pack. Announce which multiplication question (relevant to this game) produces this answer. For example, if you pick up 36 during a game practising the $5 \times$ and $6 \times$ tables, you can say that 36 is produced by $6 \times 6$ or $12 \times 3$ (but not $9 \times 4$, because that fact is not relevant to this game). You now have the choice of placing the card on top of either the number 6 or the number 12 on your paper strip. Once a card has been placed, it cannot be moved. If you cannot place the card on your turn, or if your opponent correctly challenges a mistake on your part, return the card to the bottom of the pack.
APPENDIX

Introduction to concrete manipulative materials

Manipulative materials are any materials that allow pupils to physically touch, move and rearrange them. In mathematics, they can model operations on numbers as well as the numbers themselves, and so allow learners to explore ideas, patterns and relationships in a concrete, rather than an abstract, way. Concrete materials are multisensory in that they can be appreciated by sight and by touch. They promote learning by visual, spatial and kinaesthetic routes. Teachers who ensure that all work with concrete materials is accompanied by lots of talk and discussion also cater for the auditory route to learning.

There is a variety of concrete material available to maths teachers these days, but too much variety can create problems. Presenting new models to illustrate new procedures can leave pupils with an incoherent view of maths as a series of isolated topics. For example, using a spike abacus for demonstrations of place value but for nothing else encourages pupils to compartmentalise place value thinking quite separately from thinking about mental calculations. Another common example is teaching division as repeated subtraction, but later explaining fractions through shading pictures of pizza slices, which does nothing to help pupils see the interconnection between the two concepts. This kind of fragmentation is particularly detrimental to learners with dyscalculia or those with little number sense or natural 'feel' for numbers. Pupils with difficulties benefit from having the kind of coherent model that highlights the patterns and connections within the field of mathematics.

In my view, by far the best and most versatile apparatus to use with pupils who experience difficulties with maths are continuous base-10 materials such as Cuisenaire rods and Dienes blocks. They are the most robust materials, in the sense of being capable of modelling many different situations and procedures at many different levels. Naturally, for very young children discrete materials such as counters, nuggets or cubes will precede work with rods or blocks, but overuse of discrete material tends to encourage pupils to cling to inefficient counting-in-ones strategies. Many of the activities in this book are designed to take pupils beyond such immature strategies. Working with the right concrete materials and explicitly building connections between topics help to foster a cohesive view of mathematics as a rational subject whose components are interrelated and interdependent.
Counters
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Counters do not have to be small, round plastic objects. Other examples of discrete objects that can be used as counters include cubes, wooden bricks, lolly sticks, plastic shapes, games tokens, beans, beads, pebbles, buttons, paper clips, shells, straws, Smarties, toothpicks, dried pasta shapes, bolts, washers, strips of paper, lengths of ribbon, cards, pencils, and many more. I particularly like the nuggets made of coloured or iridescent glass that Dorian Yeo introduced me to, and that are sold as flower vase fillers and table decorations.

Changing the size and shape of counters from one activity to the next can be both valuable and fun for children, larger items being more suitable for younger children and for those with dyspraxia. However, it is best to avoid too much variety of size or colour. For example, counters with solid colours are easier to distinguish and count than rainbow-coloured or patterned ones, regular shapes like discs and cubes are easier to distinguish than irregular shapes like handprints or alphabet letters, and it is easier to see the pattern of 5 if the five counters are the same size and shape as each other. Counters are very useful for work with smaller numbers but are not suitable for numbers much above 20.

Counters are particularly useful for arranging and rearranging into dot patterns. In my own teaching, I use the dice patterns for the numbers 1 to 6 and doubles or near-doubles patterns for the numbers 7 to 10. These kinds of distinctive visual patterns allow the learner to recognise and comprehend each number as a whole, so that, say, the familiar dice pattern for 5 can be instantly read as 5 without any need to count the items. By extension, a pattern showing two 5s side by side can be easily read as 10, again without any need to count. Being able to visualise, and therefore know for sure, one fact about each number provides pupils with a secure starting point from which to derive new facts through logic and reasoning.

Numicon

Numicon is a system in which flat, plastic rectangular shapes contain circular holes into which round pegs can be fitted, the number of holes corresponding to the number that is being represented. It is a popular system in many schools, partly because it is supported by so many ready-made resources.

Numicon provides separate boards for each of the numbers up to 10, with the holes regularly spaced and arranged in two parallel rows, such that all the even numbers are rectangles, while all the odd numbers have a protrusion creating an L-shape at one end. The problem with this organisation of numbers into pairs is that the larger numbers are difficult to distinguish without counting the holes. For example, 7 must be read as $2 + 2 + 2 + 1$, and is therefore easily confused with 9 which is presented as $2 + 2 + 2 + 1$. Similarly, the numbers 6, 8 and 10 are easily confused. A more significant problem is that I find the static shapes of the Numicon materials unhelpful for exploring number bonds: although they are perfect for showing the difference between odd and even numbers, they are much less successful at revealing the various ways in which numbers can be built out of, and split back into, number components.
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The Numicon board-and-peg system, confined as it is to the numbers up to 10, is no substitute for base-10 materials such as Cuisenaire rods or Dienes blocks. Unlike base-10 equipment, Numicon numbers are made of discrete items (i.e. the number 5, for example, is presented as five ones), discrete materials being far more limited in scope than continuous materials (in which the number 5, for example, is presented as one five). However, Numicon does offer a possible substitute for dot patterns; both are systems for organising quantities from 1 to 10 into a particular, fixed arrangement. Like dot patterns, Numicon pegs become cumbersome and impractical for numbers much beyond 20, and in both systems there is a danger of promoting counting strategies because the nature of the equipment emphasises the fact that all numbers are presented as a collection of ones. The limitation of discrete materials has obviously been recognised by Numicon, who now include Cuisenaire rods (which they call number rods) amongst their recommended apparatus. Because the number rods that Numicon sell are identical in every way to the rods invented by Cuisenaire, every suggestion that I make in this book that involves Cuisenaire rods can be performed, in exactly the same way, with rods supplied by Numicon.

Cuisenaire rods

Cuisenaire rods were invented by the Belgian educator Georges Cuisenaire in the 1940s to teach arithmetic to primary school children. They consist of a set of rods, made of wood or plastic, each with a cross-section measuring 1 cm square. The ten different lengths of rods start at 1 cm long and increase by increments of 1 cm. Although the largest rod is the orange 10-rod, Cuisenaire rods are compatible with Dienes blocks which provide for larger denominations. Each size of Cuisenaire rod has its own distinctive and unvarying colour, which makes its length easy to identify without the need for measuring it against unit cubes and counting the units. This is a compellingly significant characteristic and is, indeed, the main advantage of using continuous materials, rather than discrete objects. Because the rods do not carry labels or unit markings, they encourage quantities to be seen as a whole, rather than as a collection of single units. This, in turn, encourages the development of efficient calculation methods that do not depend on counting in ones.

Cuisenaire rods became very popular in the 1950s and 1960s, and were considered at the time to be a turning point in the teaching of elementary mathematics. Their unfortunate lapse into disuse had more to do with the logistical difficulties of classroom management than with any uncertainty about their intrinsic educational value, when used correctly.

I have used Cuisenaire rods in my own teaching for many years. As an aid to parents who want to support their children at home, I have produced a small leaflet of ideas that you can download from the book’s website (https://study.sagepub.com/corwin/brindoollkit4e.c5). I can warmly recommend Professor Mahesh Sharma’s various publications as well as his teacher training videos (available in the UK from Berkshire Mathematics, and in the USA from the Center for Teaching & Learning Mathematics, in Framingham, Massachusetts). Other information on using Cuisenaire rods can be obtained from the Cuisenaire Rod Company in the UK, or hand2mind (previously ETA) in the USA.

Dienes blocks

Zoltan Dienes produced his ‘attribute blocks’ not long after Cuisenaire produced his rods. The idea has been copied many times since then and the blocks are now often known generically as ‘base-10
materials. They consist of wooden or plastic blocks, all of the same colour, and are based on 1 cm cubes formed into single cubes (1), longs (10) and square flats (100). Base-10 sets nowadays also offer large cubes to represent 1000. Unlike Cuisenaire rods, base-10 materials have scored surfaces to highlight the 1 cm cube units from which they are built. Unlike the rods, there are no blocks to represent the numbers between 1 and 10, which means that the numbers below 10 have to be represented by discrete cubes and counted out one by one. For this reason, I prefer to use Cuisenaire rods for numbers up to 100, supplemented by the larger Dienes blocks for 3-digit numbers.

**Stern materials**

Stern materials were developed by Dr Catherine Stern at roughly the same time as Cuisenaire rods and are very similar in structure and purpose. Stern blocks are based on 2 cm cubes, rather than Cuisenaire’s 1 cm cross-section, which makes them satisfyingly chunky for small hands and for those with dyspraxia who may find the 1 cm rods too fiddly. Stern blocks are deliberately notched, so that they look like a series of cubes stuck together, which inevitably means that many children will count the cubes in order to find the length of the longer blocks. In order to minimise counting, each length has a fixed colour. Unfortunately, the same colours do not represent the same numbers on Stern blocks as on Cuisenaire rods, making the two systems incompatible despite their conceptual similarities.

There are some very useful and beautifully made pieces of equipment designed to go with the Stern blocks. This partly explains why Stern is so much more expensive than either Cuisenaire or Dienes. I particularly like the number boxes, especially the 10-box, the 20-tray, the number track and the dual board. Less successful in my view are the pattern boards, which were the inspiration for Numicon, in which quantities up to 10 are arranged in pairs.

Every suggestion that I make in this book that involves Cuisenaire rods can be performed just as well, and in exactly the same way, with Stern blocks. The Stern blocks’ only disadvantage – apart from the notched surfaces showing all numbers as a collection of ones, as already mentioned – is that their scale does not allow them to be combined with base-10 blocks. This limits their use to early numeracy work and to numbers below 100.

**Hybrid materials**

Bead strings, Unifix cubes and various types of abacus all attempt to bridge the divide between discrete and continuous materials. They are all useful in their own way, though none are as versatile as the combination of Cuisenaire rods and Dienes blocks.

Of the various lengths and types of bead strings, I have found the most useful to be a string of ten beads, with a colour change after five beads, because it models the all-important complement pairs that add up to 10. See Section 1 of this book for more details and for instructions for making bead strings.

Of the different kinds of abacus, I much prefer the Slavonic abacus, which is arranged as a field of 100 beads with a colour change after five beads and after five rows of beads. The colour change
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allows pupils to read quantities with a minimum of counting in ones. Ideas for using a Slavonic abacus with pupils who have difficulties with maths can be found in Eva Grauberg's work.

**How to use concrete materials**

Concrete materials should be carefully introduced and demonstrated by the teacher, who should explain to pupils what they are for, namely to make a connection between numerical magnitude and the abstract symbols we use to record numerical quantities, to make maths principles visible, and to help learners develop insight and intuition. Appropriate concrete materials allow learners to make meaning for themselves and to create a model for understanding maths that they can internalise. This cognitive model supports pupils' progression to abstract, high-level thinking.

Concrete materials are not intended to be a primitive alternative to calculating machines and should never be used in a mechanical way, simply to find an answer.

Whenever using concrete materials, it pays to be sensitive to the danger of the counting trap and to be aware of the potential for unintentionally reinforcing bad habits or inefficient calculation strategies. It is, for example, extremely important to use concrete materials in a way that discourages pupils from persistently counting in ones. For this reason, in my own teaching I always prefer: counters arranged in predictable dice or domino patterns rather than random patterns or fixed patterns that are insufficiently visually distinctive; the smooth surfaces of the coloured Cuisenaire rods as opposed to the notched or scored blocks belonging to the Stern and Dienes materials; whole rods to represent the numbers up to 10 rather than rows or collections of single cubes each representing the number 1; the Slavonic abacus in preference to either a spike abacus or an abacus with a different colour for each row of beads; bead strings that mimic the layout of the Slavonic abacus by a change in colour after every group of five in preference to bead strings constructed of many different colours of beads or with ten consecutive beads all of the same colour; number tracks that mimic the layout of the Slavonic abacus by a change in colour after every group of five spaces in preference to numbered tracks.

Concrete materials should not be used only for demonstration purposes, nor should they be used only for very basic work. They are for pupils to handle and explore, and they are most useful when the same materials are used at different stages, for different topics and at different levels of difficulty. As teachers, we must keep reminding our pupils that the actual mathematics is not what happens to numerals on paper, but what happens to numbers that are subjected to mathematical operations. Paper and pencil are just useful ways to record what happens, or to support our memory while we engage in mental calculation and abstract thinking.

In this edition of *The Dyscalculia Toolkit*, you will find more than 120 ideas for activities that propose the use of concrete materials as a route to learning and understanding maths. A small proportion of these – perhaps a tenth – is based on discrete materials, such as counters, while the rest all require continuous materials such as Cuisenaire rods and base-10 blocks. A further 100 activities and many of the games in the book are designed to help pupils' progress by leading them to make the important transition from concrete to abstract mathematical thinking.